Effects of chromatic and luminance contrast on reading

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Reading performance was measured for drifting text defined by chromatic contrast with various amounts of luminance contrast present. With 0.12 luminance contrast added, reading performance was unaffected by the presence of chromatic contrast over a range of character sizes varying 30-fold. When luminance contrast was reduced to near the threshold for reading, chromatic contrast sustained reading rates of nearly 300 words per minute, almost as high as those found with high luminance contrasts. Low-pass filtering of chromatic text had a proportionately greater effect on small characters, as would be predicted from the lower bandwidth of chromatic visual channels. Arguments are presented suggesting that reading rates for equiluminant text are sustained by luminance transients introduced by transverse chromatic aberrations of the eye.

The influence of contrast in reading is important not only because text of a wide range of contrasts is encountered in the environment but also because many ocular conditions lower the effective contrast of the reading stimulus. Most studies of the role of contrast in reading, however, have treated only the luminance dimension. In general, reading is found to be fastest when the luminance difference between text and background is maximal. For example, using high-contrast edges, Legge and Rubin1 demonstrated that for observers with normal vision the luminance contrast and the background luminance (or the text luminance when the background was dark) determine reading rate regardless of the color of the text. These results are consistent with threshold studies that show that the color of a grating does not affect luminance-contrast sensitivity2 for frequencies removed from the diffraction limit.3,4 In many situations outside the laboratory, however, reading involves text of one color with a surround of another color, in other words, with chromatic as well as luminance contrasts.

While studies have examined the interaction of chromatic and luminance contrasts on low-contrast stimuli,5-8 few have explored the suprathreshold domain. Tinker and Paterson9 found the legibility of colored inks on differently colored backgrounds to decrease with increasing color similarity. Legge and Rubin10 reported that legibility of briefly presented digits depended on the color difference between the digits and the background. While Lippert systematically varied luminance and chromatic differences between text and background, he used a range of character sizes that was near the acuity limit of the chromatic-contrast-sensitivity function. Legge et al.11 have recently reported extensive measurements of the influence of luminance and chromatic contrasts on reading, but they restricted their investigation to the long-wave limb of the CIE chromaticity diagram and examined only relatively large sizes.

To explore the relationship between chromatic-contrast sensitivity and reading, we examined reading performance for text stimuli defined by various combinations of luminance and chromatic contrasts and for characters of different sizes. We also briefly examined the influence of blurring on chromatic text. Blurring the text served two functions. First, it minimized luminance transients that might occur in the retinal image of the text owing to transverse chromatic aberrations. Second, it permitted us to assess whether the bandwidth criterion for reading chromatic text was similar to that found for luminance text. An analysis presented in Appendix A suggests that chromatic aberrations could produce substantial luminance transients under certain conditions.

METHOD

Apparatus and Calibration

All stimuli were generated by an Amiga 1000 computer and displayed on a 13-in. (33-cm) color video monitor (Amiga Model 1080). Luminances of foreground and background colors were measured directly from the monitor with a digital photometer (Spectra). The chromaticity coordinates of all stimuli were measured with a spectrophotometer with tristimulus filters (Photo Research), which was also used to check the luminance, and were subsequently checked with a spectroradiometer (EG&G).

Stimuli

In the following discussion λ and μ will be assumed to represent lights of different spectral composition that, additionally, map into distinct coordinates in the CIE chromaticity diagram. The luminance contrast of all text stimuli will be specified by using Michelson's formula,12 regardless of the color difference between text and background. The presence of chromatic contrast in a text–background pair will be defined as there being a difference in their chromaticity coordinates, but no further attempt will be made to quantify this contrast since it depends in a complex manner on both the location and the distance between the two points in the space. Quantifying the chromatic contrast within a uniform chromaticity diagram would also be of limited use, as such diagrams that are available are uniform only over local re-
Knoblauch et al.


Table 1. Summary of Chromaticity, Luminance, and Contrast Characteristics of Text Stimuli

<table>
<thead>
<tr>
<th>Stimulus</th>
<th>x</th>
<th>y</th>
<th>$Y_{\text{high}}$</th>
<th>$Y_{\text{low}}$</th>
<th>$C_{\text{high}}$</th>
<th>$C_{\text{low}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.31</td>
<td>0.35</td>
<td>37.4</td>
<td>1.06</td>
<td>0.956</td>
<td>0.128</td>
</tr>
<tr>
<td>Y</td>
<td>0.41</td>
<td>0.55</td>
<td>47.4</td>
<td>35.3</td>
<td>0.157</td>
<td>0.010</td>
</tr>
<tr>
<td>BG</td>
<td>0.25</td>
<td>0.35</td>
<td>47.5</td>
<td>36.5</td>
<td>0.140</td>
<td>0.009</td>
</tr>
<tr>
<td>M</td>
<td>0.32</td>
<td>0.26</td>
<td>49.1</td>
<td>37.7</td>
<td>0.124</td>
<td>0.007</td>
</tr>
</tbody>
</table>

*All values above describe only the text characters. Contrasts were computed with respect to a white background of 48.4 cd/m². The $Y_{\text{high}}$ luminances were used for computing the $C_{\text{high}}$ contrasts, and the $Y_{\text{low}}$ luminances were used for computing the $C_{\text{low}}$ contrasts.*

Fig. 1. Luminance profiles of contours of characters illustrating various combinations of luminance and chromatic contrasts. 1, A dark contour embedded in a background of spectral composition $\mu$ illustrates maximum luminance contrast ($L$). 2, A contour of spectral composition $\lambda$ is matched in luminance to a background of different spectral composition and of different chromaticity, illustrating chromatic contrast ($C$) with no luminance contrast. 3, A contour of the same spectral composition as the background illustrates an intermediate value of luminance contrast with no chromatic contrast. 4, A contour matched in luminance contrast with case 3 but differing in chromatic contrast illustrates a stimulus with luminance and chromatic contrast ($L + C$).

Fig. 2. Color specification of the text and background stimuli used in this study represented in the CIE chromaticity diagram. The solid triangle indicates the color gamut measured on the video display. The points indicate nominal colors W (white), Y (yellow), BG (blue-green), and M (magenta), whose chromaticity coordinates and luminances are indicated in Table 1.

We used a fixed-space, sans-serif font. Center-to-center spacing of a pair of the capital letter O was 1.1 cm, and the height was 2 cm. Reading material was obtained from stan-

&nbs
standardized reading-comprehension tests and corresponded to ninth-grade reading levels.

The influence of chromatic aberration was examined by blurring the text with a sheet of ground glass (1.4-mm thickness, Edmond Scientific) interposed between the observer and the display. Ground glass has been shown in a number of contexts to function as a spatially isotropic, low-pass filter with an approximately Gaussian shape. The distance between the ground glass and the display was chosen so that the bandwidth of the filter (measured by the point at which its modulation transfer function had declined to 1/e) was 2 cycles per character. Over a wide range of character sizes, Legge et al. found this value to be the minimum-spatial-frequency bandwidth required for nearly optimal reading. For our apparatus, this distance was determined by the following procedure. First, a vertical sine-wave grating of 1.82 cycles per centimeter (or 2 cycles per width of an individual character) was presented to an observer on a cathode-ray tube (Joyce Electronics, P4 phosphor). The mean luminance was set at 50 cd/m², nearly the same as it was in the main experiment. In principle, the spatial frequency at which these measurements were made can be arbitrary as long as it is resolvable. The observer sat at a distance so that the grating subtended 3 cycles per degree, as this frequency corresponds roughly to the peak of the contrast-sensitivity function. While the grating was presented continuously, the observer made ten threshold settings. The contrast of the grating was then raised to a value e times the mean of the ten threshold values. At this point, the ground glass was introduced between the display and the observer, and the distance determined from the display that reduced the grating again to the threshold (6 mm). This distance was used in experiments in which it was desired that the text be blurred with a spatial filter of bandwidth 2 cycles per character.

The spectral-filtering characteristics of the ground glass were checked by having an observer make heterochromatic flicker-photometric matches on a color display (Electrohome, G09-401) with and without the ground glass interposed between the observer and the display. The stimulus was a disk of 1-deg diameter presented on a dark surround at a luminance of 50 cd/m². Two of the guns were alternated temporally in square-wave fashion at 12.5 cycles per second. The presence of the ground glass was not found to produce a significant difference in the photometric settings for any of the pairings of the guns of the display.

Observers
Two of the authors served as observers for all experiments. Each had normal color vision and normal or corrected-to-normal visual acuity. The uncorrected distance acuities of the subjects were 20/10 for observer AA and 20/20 for JS. The main results were replicated on naïve observers.

Procedure
Observers received extensive practice on the reading task during collection of pilot data for this and other projects. Observers were light adapted to the screen luminance for 5 min before each session. One color contrast was tested in a given session, and the size of the text was randomly varied within sessions. Color contrast was varied randomly across sessions.

Each experimental trial consisted of a presentation of a line of 80 characters of left- and right-justified text (spaces included) drifted at fixed velocity from right to left through a window of 6-character width located in the middle of the screen. The reduced window size confined the text to a relatively homogeneous portion of the screen. Legge et al. have shown that optimal reading performance with drifting text is obtained when the window is larger than a 4-character width. Preliminary data collected with a 45-character win-
The observer tracked the text monocularly and read it aloud. Text velocity was increased until the observer could not read the text without making errors. The minimum speed for which this was true, corrected for the number of errors, was taken as an estimate of reading rate. Three measurements of reading rate were obtained for each character size within each session.

Character size was varied by changing the viewing distance. Trial lenses aided accommodation at close distances. Character width varied between 0.2 and 6.0 deg of visual angle.

**RESULTS**

Figure 3 shows data from two observers, measuring reading rate as a function of character width for text that varied only in luminance contrast. Each point represents the geometric mean of three measurements. At high contrast, reading rate varied only slightly for character widths between 0.2 and 6 deg (filled circles). Reading rate peaked at character widths between 0.2 and 1 deg and fell off gradually from this peak out to character widths as large as 6 deg. Lowering the luminance contrast to 0.12, approximately a factor of 7 (open circles), reduced reading rates by an average of 0.1 log unit for the two observers. This small difference, however, was not statistically significant ($F_{1,1} = 29.13; p = 0.012$). Lowering the contrast to 0.02 reduced reading rates more dramatically, with the largest effects at the smallest character widths (filled squares). The peak reading rate shifted to character widths between 2 and 4 deg for low contrasts. In accord with the results of Legge et al., we found that reading performance in observers with normal vision was immune to luminance-contrast reduction over a large range.

Figure 4 shows, for two observers, a comparison of reading rates in the presence and the absence of chromatic contrast for conditions in which luminance contrast is approximately matched. The results for the achromatic text are replotted from Fig. 3 as open circles and represent performance for a luminance contrast of 0.12. The nominal hue of the chromatic text is designated in the figure for each symbol type. Each had a similar luminance contrast (see Table 1). For observer AA, the presence of chromatic contrast produced a small but systematic improvement in reading rates at the three largest character widths tested, but no similar effect was evident in the data of JS. The logarithm of the ratio of reading rates between chromatic-plus-achromatic text and achromatic text is plotted in Fig. 5 as a function of character size. Points above the baseline indicate improvement of reading performance due to the presence of chromatic contrast. Statistical tests did not reveal a significant effect of chromatic contrast on reading performance, either when the four conditions were compared as in Fig. 4 ($F_{2,2} = 2.57; p = 0.23$) or when the difference from the achromatic condition was evaluated as in Fig. 5 ($F_{2,2} = 2.50; p = 0.28$).

Figure 6 examines reading performance when the text contrast was near equiluminant. Recall that the residual luminance contrast for each of the chromatic curves was less than or equal to 0.01 (see Table 1). For purposes of comparison, the achromatic-contrast curves from Fig. 3 for 0.96 and 0.02 contrast have been reproduced as filled circles and squares, respectively. All the near-equiluminant curves fell between these two curves. Although at the largest character width some colors (Y for JS and Y and BG for AA) produced reading nearly as fast as high-luminance-contrast text, reading performance with near-equiluminant text was not better than high-contrast luminance text at any character width ($F_{2,2} = 31.12; p = 0.03$). Except for three points in the data of observer AA, the near-equiluminant curves were all above the low-contrast achromatic curve. Since there was less
luminance contrast present in the near-equiluminant curves than in the low-contrast achromatic curve and since the residual luminance contrast, if present alone, would be nearly at or below threshold, the increment in reading performance must have been due to the presence of the chromatic contrast. Interestingly, the greatest increases in performance for the chromatic conditions were at the smallest character widths. For both observers, the ordering of reading performance from worst to best was M, Y, and BG.

Figure 7 shows, for observer AA only, reading rates as a function of character size for near-equiluminant text passed through a spatial filter of 2 cycles per character bandwidth. Reading rate declined sharply for characters smaller than 0.7 deg. For larger characters, the relative change in reading performance was consistent with that shown in Fig. 6 for this range. The ordering of the colors was the same as that shown in Fig. 6 except for the reversal between Y and BG at 3 deg. The filtering by the ground glass produced the most
DISCUSSION

The use of oral reading for assessing the capacity of the visual system for processing information raises the question of whether speech was the limiting factor in these measurements. While it is difficult to prove otherwise for character sizes at the peak of the high-contrast achromatic function, for which reading speeds reached 300–500 words per minute, speech cannot be the limiting factor off the peak to either side for which reading speed declines. It should be emphasized that at no time did observers ever receive a 300-word trial, as each 80-character line averaged approximately 12 words in length. The short line length permitted observers to recite a portion of the trial from memory. Our preliminary data\(^\text{18}\) and that of others\(^\text{11,13}\) who have utilized silent controls indicate that somewhat higher reading rates can be obtained with silent reading, but the forms and relations of the functions do not change. Unfortunately, with silent reading, error rates are not easily assessed. Higher reading rates have also been obtained by using shorter lines.\(^\text{20}\)

Our reading rates are not exceptional for these conditions. References 11 and 13, with similar conditions, regularly reported peak reading rates of at least 300 words per minute, and Legge et al.\(^\text{11}\) have recently reported an observer who read at rates as high as 670 words per minute. Anecdotally, observers often perceived themselves as speech limited when they were not. For example, for 2-deg high-contrast achromatic letters, observer AA complained that his reading speeds were limited by his ability to speak fast, but, when character size was reduced to 0.3 deg, his reading performance increased 67%.

Our results show that, for observers with normal spatial and color vision, the effect of chromatic contrast on reading performance depends on the level of luminance contrast present. For luminance contrasts that produced only a modest decrement in reading performance, the introduction of chromatic contrast produced no systematic effect on reading rate. For example, we did not see an improvement in reading performance with chromatic contrast added to a luminance contrast of 0.12. We emphasize that these results are not likely to reflect a ceiling effect on oral performance at character sizes off the peak, where reading speed has declined. These results are consistent with those of Legge et al.,\(^\text{11}\) who found luminance and chromatic contrasts to combine independently in determining reading speed.

When luminance contrast was sufficiently lowered, effects of chromatic contrast were evident. In general, chromatic text with luminance contrast less than 0.01 produced higher reading rates than achromatic text with 0.02 contrast. The improvement in performance was greatest for the smallest character sizes. Reading performance with near-equiluminant text was quite high, reaching nearly 300 words per minute, though not better than that for luminance text of high contrast at any character width. It is important to point out, however, that higher chromatic contrasts could have been generated by using a background other than W (e.g., Y against B). Thus rates higher than those measured here might be attainable.\(^\text{11}\)

Lippert\(^\text{10}\) found that legibility of digits as measured with a reaction-time task could be predicted with a color-difference metric based on a modification of CIE 1976 (\(L^*, u^*, v^*\))...
uniform color space. He presented a nomogram for assessing the legibility of character-background pairs for characters in the size range 0.36-0.55 deg. To evaluate Lippert's result and our own, we must consider the constraints placed on reading rate by contrast sensitivity with respect to character size.

Recent estimates of the contrast sensitivity for chromatic gratings place the high-frequency cutoff at between 10 and 12 cycles per degree. Assuming letters that are the dimension of standard Sloan optotypes used in acuity testing (5 times as wide and high as the stroke width), these values set a limit of 0.21 to 0.25 deg on the minimum size for resolving the chromatic spatial variation of letters. The font used in the present study was approximately twice as high as it was wide so that observers might be expected to have a somewhat lower letter-size limit. Nevertheless, reading rate for near-equiluminant text began to fall for character widths below approximately 0.3 deg.

Legge et al. have shown that a minimum bandwidth of 2 cycles per character is necessary for sustaining the maximum reading rate of text defined by luminance contours. This value is based on the 1/e decay point of low-pass spatial characteristics that these investigators used for filtering their text stimuli. From Mullen's data for chromatic contrast sensitivity, the 1/e bandwidth is approximately 3 cycles per degree for both R-G and Y-B chromatic-contrast-sensitivity functions. By this criterion, reading rate should decline for equiluminant text with characters smaller than 0.67 deg. This argument depends on the assumptions that the chromatic-contrast-sensitivity function behaves as a low-pass filter and that, as with luminance text, observers require at least 2 cycles per character for reading chromatic text, optimally. By the above arguments, the size range that Lippert examined would seem to be below what is optimal for mechanisms sensitive to chromatic contrast. We examined characters in the range 0.2-6.0 deg wide and found no evidence to suggest a rolloff in performance until well below 0.67 deg. Thus, at first glance, reading performances with near-equiluminant text seem to be superior to those that would be predicted by the constraints of chromatic contrast sensitivity.

We can think of two possible explanations for the high reading rates for small characters defined by near-equiluminant contrast. Throughout this study luminance was defined in terms of photometer readings based on the CIE standard observer. Individual deviations from the standard observer may have introduced luminance cues that provided sufficient contrast in order to support reading in the presence of unresolved chromatic contours. The coarsely sampled color gamut of the Amiga color space, unfortunately,
prevented us from using individual corrections. Nevertheless, both observers performed similarly with respect to all conditions examined, as have several other, naïve subjects used in pilot experiments.

A second possible source of luminance cues is chromatic aberrations of our observers' eyes. Transverse (or lateral) chromatic aberrations can produce a wavelength-dependent spatial shift in the retinal image. This shift is a prismatic effect that results from the lack of correspondence between the optical and visual axes of the eye. The effect of the shift on luminance edges defined by broadband light is to reduce the contrast of high-spatial-frequency components. Indeed, this phenomenon has been held to be the primary factor limiting luminance-contrast sensitivity near the acuity limit in broadband light. With chromatic stimuli, however, spatial luminance transients are usually introduced through the spatial skewing of the spectral distributions on the two sides of an edge (Fig. 8).

Figures 9 and 10 show predicted luminance profiles after the introduction of transverse chromatic aberrations for character strokes from various width fonts. (See Appendix A for details of calculations.) The stroke in each case was defined by a spectral distribution $\lambda$ taken from one of the three near-equiluminant text conditions of the current study in a surround of $W$ light, $\mu$ (symbols and edge locations are indicated only in the top left-hand boxes of Figs. 9 and 10). The skewing of $\lambda$ with respect to $\mu$ results in a decremental transient on one edge and an incremental one at the other. These spatial transients would outline each character, increasing its effective luminance contrast. The blurring effects from axial chromatic aberrations and the line spread of the eye have not been included in these simulations in order to emphasize specifically the prismatic effects. While the line spread of the eye would be expected to lower the effective luminance contrast, our calculations (Appendix A and Table 2) indicate these transients to be more effective than threshold stimuli. The figures show that, as stroke width is reduced, the distance between the luminance transients decreases. At a sufficiently small character width, the stroke would be defined almost entirely in terms of luminance contrast.

Recently Thibos, et al. have measured an average value of 2.0 deg for the average offset between the optic and visual axes.

### Table 2. Characteristics of Luminance Transients Generated by Lateral Chromatic Aberrations

<table>
<thead>
<tr>
<th>Eccentricity (deg)</th>
<th>Half-Width at Half-Height (sec)</th>
<th>Peak Deviation from Mean (cd/m²)</th>
<th>Estimated Visibility Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y/W</td>
<td>19.0</td>
<td>12.9</td>
<td>21.6</td>
</tr>
<tr>
<td>M/W</td>
<td>9.4</td>
<td>6.5</td>
<td>4.9</td>
</tr>
<tr>
<td>BG/W</td>
<td>10.3</td>
<td>12.9</td>
<td>12.2</td>
</tr>
<tr>
<td>R/G</td>
<td>10.0</td>
<td>5.0</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Fig. 10. Same as Fig. 9 except that the offset of the optic from the visual axis is 2.0 deg.
axes. For the simulations in Figs. 9 and 10, however, we used values of 5.0 and 2.0 deg, respectively, because previous measures\textsuperscript{24} gave a value of approximately 5.0 deg. With the broadband sources produced by our display, the transient at the edges of each letter would be a combination of the wavelength spectra of the text and background smeared across the transient and weighted individually by the luminances at each wavelength. The acuity limit of the visual system would not be expected to permit resolution of the spectrum within the transient, except perhaps as a fringe at the edge. Under natural viewing conditions, however, it is thought that the fringes are not typically perceived as colored, owing to adaptation.\textsuperscript{25}

We specifically avoided using an achromatizing lens that might lessen such aberrations because such a lens would not be present for observers in the real world for situations in which they encountered text defined by color contrast. Additionally, the task requirement that observers track drifting text while responding orally would have made the control of chromatic aberrations with such a lens or an artificial pupil impractical. However, to evaluate the potential influence of chromatic aberrations in our data, we examined reading performance with text blurred by a spatial filter of 2 cycles per character bandwidth. While such a filter does not completely eliminate the luminance contrast in the retinal image, it attenuates the high spatial frequencies in the stimulus that would produce the highest-contrast luminance components. (See Appendix B.) Without these high spatial frequencies contributing to luminance artifact, reading rate appears to be constrained by the bandwidth of the chromatic-contrast-sensitivity function as evidenced by the drop in reading performance for characters smaller than 0.67 deg, indicated by the dashed line in Fig. 7. The lower overall rate may reflect the true performance of the chromatic systems for reading. Thus, on one hand, the sharp decline in reading performance for spatially filtered, near-equiluminant text supports the notion that chromatic-contrast sensitivity constrains reading of equiluminant text according to the same principles as luminance-contrast sensitivity. On the other hand, the high reading rates for unfiltered, near-equiluminant small characters raise the intriguing possibility that in practical situations chromatic aberrations may have the beneficial effect of enhancing the visibility of some contours that would otherwise not be resolvable.

APPENDIX A: TRANSVERSE CHROMATIC ABERRATIONS OF BROADBAND APERIODIC STIMULI

The purpose of this appendix is to evaluate computationally, for the stimuli that were used in the present study, whether the luminance transients at chromatic edges in the retinal image generated by transverse chromatic aberrations of the eye were sufficiently visible to influence reading performance. Only the effects of transverse chromatic aberrations are considered here. Diffraction and other aberrations have been excluded from the analysis, although longitudinal chromatic aberrations, as well, may introduce frequency-dependent luminance components in equiluminant stimuli.\textsuperscript{26} Our analysis considers only the effects on edges that are orthogonal to the line between the optical and visual axes, which should be nearly vertical, even though smaller effects can be expected from oblique edges as well.

Thibos\textsuperscript{22} presented a method based on a reduced-eye model for computing the luminance distributions resulting from transverse chromatic aberrations in the retinal image of a broadband (or polychromatic) sine-wave grating. The grating is computed to be the integral over wavelength of the luminance distribution with a phase-shift term that depends on wavelength. The method can be adapted for computing the luminance distribution of aperiodic images of arbitrary spectral composition.

The horizontal luminance profile \(L\) of a two-color image can be considered to be the integral over the wavelength of the sum of foreground \((F_g)\), background \((B_g)\), spatial \((x)\) and spectral \((\lambda)\) luminance distributions:

\[
L(x) = \int_{\lambda_0}^{\lambda_f} [F_g(x, \lambda) + B_g(x, \lambda)] \, d\lambda.
\]

The effect of transverse chromatic aberrations is to convolve the foreground and background distributions with a delta function whose spatial offset in degrees is wavelength dependent:

\[
L(x) = \int_{\lambda_0}^{\lambda_f} [F_g(x, \lambda) + B_g(x, \lambda) \ast \delta(x - \phi(\lambda))] \, d\lambda.
\]

The function \(\phi(\lambda)\) in the model of Thibos\textsuperscript{22} is given by

\[
\phi(\lambda) = \epsilon - \alpha + \beta,
\]

where \(\epsilon\) in the present circumstances is the angular difference between the optical and visual axes

\[
\alpha = \sin^{-1}[3.98 \sin(\epsilon)/5.55],
\]

\[
\beta = \sin^{-1}[\sin(\alpha)/n]
\]

and \(n\) is taken to be the index of refraction of the model eye based on Cornu's formula for water:

\[
n = 1.31848 + 6.662/(\lambda - 129.2),
\]

where \(\lambda\) is measured in nanometers. (Consult Thibos\textsuperscript{22} for a detailed discussion of the parameters \(\alpha, \beta, \epsilon\).) The wavelength dependence of \(\phi\) results from the formula for \(n\). The symbol \(\ast\) represents the convolution operation.

The effect of transverse chromatic aberrations can be visualized as a skewing of the spectral distribution of luminance in the \(\lambda-x\) plane. The results of such a skewing are shown schematically as the dotted curves in Fig. 8 for a Y bar on a W background. Since luminance at a given position is computed by integrating across the wavelength, the luminance in the vicinity of a chromatic edge is composed of components of the spectral distributions from both sides of the edge. If the foreground and the background are set to be equiluminant in the object plane, then incremental or decremental luminance transients will result at the edge in the retinal image.

Horizontal luminance profiles for foveally viewed letter strokes that include transients generated by the transverse component of chromatic aberrations are shown in Figs. 9 and 10 for letter sizes and color contrasts corresponding to those used in the present study, under two different assumptions about the angular difference between the visual and optical axes. These profiles were computed as follows. The spectral energy distributions of the R, G, and B guns of our...
display were measured with an EG&G spectroradiometer. The chromaticity coordinates of these were calculated. Given the chromaticity coordinates and luminance of each of the colors that we wished to check (Table 1), the tristimulus values were computed, and the quantities of the gun primaries \((k_R, k_G, k_B)\) for producing each of these colors were calculated from the system of equations

\[
\begin{bmatrix}
X_R & X_G & X_B \\
Y_R & Y_G & Y_B \\
Z_R & Z_G & Z_B
\end{bmatrix}
\begin{bmatrix}
k_R \\
k_G \\
k_B
\end{bmatrix}
= \begin{bmatrix}
X_C \\
Y_C \\
Z_C
\end{bmatrix},
\]

(A4)

where the columns of the matrix are the tristimulus values for each of the guns, R, G, and B, and the vector on the right-hand side of the equation represents the tristimulus values for an arbitrary light \(C\). The coefficient vector of \(k\)'s was used to calculate a linear combination of the spectral energy distributions of the gun primaries for generating the spectral distribution of \(C\). This spectral energy distribution was then weighted by Judd's modified \(V(X)\) function and normalized to sum to the appropriate luminance

\[
L_C(\lambda) = tV(\lambda)[k_R R(\lambda) + k_G G(\lambda) + k_B B(\lambda)],
\]

(A5)

where \(t = Y/L_\sum_\lambda V(\lambda)[k_R R(\lambda) + k_G G(\lambda) + k_B B(\lambda)]\) will be called the spectral luminance distribution of the light \(C\).

The skewing function \(\phi(\lambda)\) was computed for eccentricities of 5 and 2 deg at intervals of 10 nm. While earlier studies of chromatic aberration yielded values for the eccentricity of the optic axis of approximately 5 deg,\(^{22}\) more recent studies\(^{23,25}\) indicate a smaller value, perhaps as small as 2 deg. For convenience, the skewing function was normalized to produce zero shift at 700 nm by subtracting \(\phi(700)\) from it.

If \(L_L(\lambda)\) and \(L_R(\lambda)\) are the spectral luminance distributions on the left-hand and right-hand sides of an edge at position \(x_0\), then the luminance, \(L(x)\), at position \(x\) can be computed with the algorithm

\[
\text{if } [x < x_0 + \phi(\lambda)],
\]

then \(L(x) = L(x) + L_L(\lambda),\)

else \(L(x) = L(x) + L_R(\lambda),\)

(A6)

evaluated iteratively over \(\lambda.\)

Figures 9 and 10 show the luminance profiles resulting from our computations for 5- and 2-deg eccentricities, respectively, of the optical axis. The luminance transients appear as narrow spikes, incremental on one edge and decremental on the adjacent one. They vary in peak deviation from the mean luminance and width at half-height according to the color pairs utilized (see Table 2). Reducing the size of the letter simply brings the opposite edges closer together and, in so doing, brings the transients closer in proximity. It is possible that for the smallest characters the proximity of the transients permits them to interact to produce effectively an even greater local luminance contrast.

Reducing the eccentricity of the optical axis from 5.0 to 2.0 deg reduces the half-width at half-height of the transients by, on average, a factor of 2.4 but leaves the peak deviation unchanged.

An additional computation included in Table 2, in the R/G row, is for a R character stroke against a G background at 6.0 cd/m². This computation was performed simply by using the R and G guns alone at equal luminance for foreground and background, respectively. This condition was included because it corresponds to one used by Legge et al.\(^{11}\) in their study of color contrast and reading. While strong effects of chromatic aberrations are usually associated with juxtaposition of extreme portions of the spectrum, it is noteworthy that the R/G combination produces a luminance transient with a peak deviation of greater than 80% of the mean luminance. The reason for this extreme effect is undoubtedly the narrow-band line at approximately 620 nm that is typical of the spectral energy distribution of the R primary of television displays. Thus, while this calculation was based on the R and G primaries from the current study, the conclusions are likely to hold for other display systems as well. Legge et al.\(^{11}\) assessed the axial chromatic aberration of their stimuli and found it to be minimal, but they did not evaluate the transverse component.

Visibility of these transients was estimated by comparison with data from Hecht and Mintz,\(^{29}\) who measured the minimum width of dark lines visible as a function of luminance level. They demonstrated that resolution of thin lines depends on intensity discrimination rather than on acuity. Under optimal conditions, observers were found to be able to detect lines of 0.5-sec width and 3.0-deg length. We can compare their results with our decremental transients because the transients are sufficiently narrow that, owing to the line spread of the eye, their retinal distribution should be independent of size.

Hecht and Mintz found that observers could resolve lines of approximately 0.5- and 0.75-sec width, respectively, at 50 and 6 cd/m². We assume that the background had the spectral distribution of illuminant A.\(^{30}\) The luminance profile of each of these line widths was computed by assuming an eccentricity of either 5.0 or 2.0 deg from the optic axis, from the second equation of Note 28, and the areas of the decremental profiles were computed. Similarly, the areas of the decremental profiles of the decremental transients of Figs. 9 and 10 were calculated. The ratios of the areas of the transients from Figs. 9 and 10 to the corresponding areas from the line-resolution data were taken as estimates of the factor above threshold (if greater than 1.0) of the decremental luminance transients. These factors are displayed in the Estimated Visibility Factor column of Table 2.

All the factors are greater than 1.0, indicating that the transients are all above threshold. Even with the assumption that the difference between the optic and visual axes is as small as 2.0 deg, the luminance transients arising at the chromatic contour are predicted to be substantially above threshold for some of the color pairs. Among the stimuli used in the current study, the predicted effectiveness of the transient from the BG/W condition may account for the substantial advantage for reading that this condition displayed over the other near-equiluminant conditions in Fig. 6. Why the Y/W condition should be superior to the M/W condition, which displays more salient luminance transients by the current criterion, is not known. It should be indicated that the Y/W stimulus has a greater colorimetric purity (88.2% versus 42.2%) and a greater residual luminance contrast (0.01 versus 0.007) than the M/W stimulus. We cannot exclude these or other factors as the possible sources of the superiority of the Y/W over the M/W condition.
APPENDIX B: SPATIAL-FREQUENCY CHARACTERISTICS OF TRANSIENTS FROM TRANSVERSE CHROMATIC ABERRATIONS

The luminance transients in the retinal image that result from lateral chromatic aberrations contribute most of their energy in the high spatial frequencies and outside the minimum bandwidth required for reading. The first part of this assertion can be seen from considering that the transients are localized to edges of the contours. However, the arguments for both parts can be made quantitative. Consider that the horizontal luminance profile \( L(x) \) of the pair of transients that define the two edges of an isolated contour of an equiluminant character stroke of width \( x_0 \) is approximately of the form

\[
L(x) = f(x/w) \ast \left[ \delta(x + x_0/2) - \delta(x - x_0/2) \right],
\]

where \( f(x) \) is the luminance profile of the transient at a single edge, \( w \) is a scaling factor that determines the width of the transient, \( \ast \) represents the convolution operation, and we have suppressed a term for the mean luminance level. The Fourier amplitude spectrum of the above expression is

\[
\mathcal{L}(v) = |w|F(w) \sin(\pi x_0 v),
\]

where \( F \) is the Fourier transform of \( f \) and \( v \) is spatial frequency.

Given unimodal \( f \) and values of \( w \) in the range 0.003–0.006 deg (see Table 2 and Figs. 9 and 10), we can assume that \( F \) will be approximately constant over the visible range of frequencies. Thus the frequency dependence of \( \mathcal{L} \) is sinusoidal. \( \mathcal{L} \) is zero at \( v = 0 \) and peaks at multiples of \( 1/(2x_0) \).

Recall from the Discussion that the reading speed of luminance text is undisturbed if a bandwidth of at least 2 cycles per character is provided. In terms of standard Sloan optotypes, the minimum bandwidth would be \( 2/(5x_0) \), or 20% below the peak of the first mode of \( \mathcal{L} \). If the bandwidth requirement for reading chromatic text are similar to that of luminance text, then, clearly, a considerable portion of the spectrum of \( \mathcal{L} \) can be removed by low-pass filtering without disturbing the minimal information present in the character for optimal reading. Note also that \( \mathcal{L} \) scales with letter size, so that one cannot argue that chromatic aberrations will be less of a problem with large letters in the way in which it can be so argued for low spatial frequencies. This point is evident from Figs. 9 and 10 also.

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REFERENCES AND NOTES


12. Michelson's formula specifies contrast as \( (L_{\text{max}} - L_{\text{min}})/L_{\text{max}} \), where \( L_{\text{max}} \) is the maximum luminance and \( L_{\text{min}} \) is the minimum. While this formula is appropriately applied to periodic stimuli, it also has been used in other contexts for specification of reading stimuli (e.g., Refs. 11, 13, and 14). We utilize it here for convenience and so that our results can be easily compared with other reading studies that have specified their stimuli similarly. Strictly speaking, the contrasts specified here are not directly comparable with those given for periodic stimuli, such as sine-wave gratings.


15. Given the maximum luminance available from each of the guns of our display, these lights were located at the limits of the equiluminant color gamut at this luminance level. The colorimetric purities of Y, BG, and M were estimated to be, respectively, 82.2%, 27.0%, and 42.2%, based on W. Dominant wavelength lengths were estimated as, respectively, 567 nm, 493 nm, and 551 nm.


19. For distances closer than 2 m, observers were fitted with a trial lens with dioptic power equal to the reciprocal of the desired distance to the display. Observers were then positioned so that the lens was one focal length from the display. Such an arrangement produced a unit magnification independent of the distance to the observer's eye behind the lens. Because the observer's head position was free, the distance to the display was checked freely throughout an experimental session, and trials were discarded if the distance of the subject to the display was found to have changed.


28. Note that this algorithm will produce spurious results if the width of a contour is smaller than the variation of \( \phi \), owing to the use of discrete wavelength intervals. This was not the case for the character sizes considered here. However, if it were, one could utilize the formula

\[
L(x) = \int_{0}^{700} Bg(\lambda) d\lambda + \int_{h_{x}}^{700} Bg(\lambda) d\lambda + \int_{h_{x} - \omega}^{h_{x}} Fg(\lambda) d\lambda,
\]

where \( \theta(x) = \phi^{-1}(\lambda) \), the inverse of \( \phi \), and \( w \) is the width of the contour. It is convenient to sample the spectral luminance distributions and the function \( \theta(x) \) more densely with an interpolating function, such as a cubic spline. The additional accuracy of the above equation was not found to produce significant changes in the luminance profiles calculated for character strokes reported here. If \( Fg(\lambda) \) is zero, then the above equation reduces to

\[
L(x) = \int_{0}^{700} Bg(\lambda) d\lambda - \int_{h_{x} - \omega}^{h_{x}} Bg(\lambda) d\lambda.
\]
