

On Quantifying the Bipolarity and Axis of Farnsworth-Munsell 100-Hue Test

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A simple method is introduced for analyzing error score distributions from the Farnsworth-Munsell 100-Hue test. The method involves fitting a sine wave directly to the error distribution, and produces estimates of three parameters that characterize the severity of the defect, the degree of bipolarity, and the orientation of the axis of bipolarity, if one is present. The method produces good estimates of the orientation of the axes for congenital defects. It is also amenable to statistical analysis. Invest Ophthalmol Vis Sci 28:707-710, 1987

The Farnsworth-Munsell 100-Hue test (FM 100-Hue) is widely used in clinical, industrial, and experimental settings for evaluating color vision discrimination.^{1,2} Interpretation of the results of this test depends on 1) the total error score, 2) the degree to which errors are distributed along a particular axis on the score sheet, and 3) the orientation of that axis, if it is present. Typically, the only measure that is quantified is the total error score. The degree to which a particular axis of error (bipolarity) is present and its orientation are usually evaluated by eye. Recently, several techniques have been introduced to evaluate quantitatively the axis of the error distribution.^{3,4}

For example, Kitahara's³ technique involves a type of filtering (smoothing), followed by normalization of the error distribution. When an axis is present, this technique reveals a sine wave-like pattern of 2 cycles. Kitahara indicates that the amplitude and phase of a sine wave fit to this pattern determine, respectively, the degree of bipolarity and the orientation of the axis of the color defect.

There is a more direct approach to extracting the degree of bipolarity and its orientation. Following Kitahara, it is assumed that this information is contained in the sinusoidal component of the error distribution at a frequency of 2 cycles/85 caps (ie, one revolution around the error diagram). It is not necessary, however, to transform the error distribution prior to extracting the amplitude and phase of this component. For sine waves, the values of the parameters from a least-squares

fit applied directly to the error distribution yield the amplitude and phase of that sinusoidal component present in the distribution.⁵ Thus, the proposed method is to fit the error distribution with an expression that is the sum of a constant term and a sinusoidal term of appropriate amplitude and phase. This procedure yields estimates of three parameters that characterize 1) the overall error, 2) the degree of bipolarity, and 3) the orientation of the axis of the bipolarity.

Materials and Methods

Three parameters are estimated from the distribution of cap scores: the mean error and the amplitude and phase of a sine wave of two cycles per revolution around the error diagram. The function fit to an error pattern is:

$$f(i) = M + A \cdot \sin(4\pi(i-1)/85 + \phi), \quad (1)$$

where i is cap position, M is the mean error, A is the amplitude of the sine wave, and ϕ is the phase angle of the sine wave.

The mean error, M , is computed using the following formula:

$$M = \sum_{i=1}^{85} e_i / 85 \quad (2)$$

where e_i is the cap score associated with position i , according to either the Farnsworth⁶ or Kinnear⁷ methods of plotting.*

* The caps are numbered consecutively underneath. After a patient has ordered them, the cap score of cap i is computed from the sum of the absolute values of the difference between cap i and its two adjacent caps. According to the Farnsworth method,⁶ this cap score is assigned to the position on the error diagram associated with the number under cap i . According to the Kinnear method, this cap score is assigned to the position occupied by cap i . Both methods usually produce similar error distributions.¹

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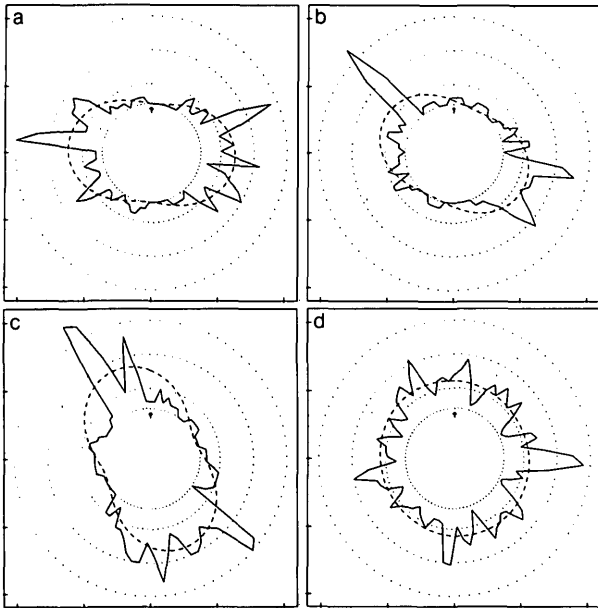


Fig. 1. Error distributions for four color defective observers. Tests were administered under illuminant C at approximately 200 lux. a. protanope b. deuteranope c. tritanope d. achromat.

The amplitude and phase angle are determined separately by first computing the amplitude of sine and cosine components:

$$a_s = \sum_{i=1}^{85} e_i \sin(4\pi(i-1)/85)/42.5 \quad (3a)$$

$$a_c = \sum_{i=1}^{85} e_i \cos(4\pi(i-1)/85)/42.5, \quad (3b)$$

where a_s , a_c are the sine and cosine amplitudes, respectively, and the arguments of the trigonometric functions are in units of radians.

Then, the amplitude and phase are, respectively:

$$A = (a_s^2 + a_c^2)^{1/2} \quad (4)$$

$$\phi = \begin{cases} \tan^{-1}(a_c/a_s), & a_s > 0 \\ \pi + \tan^{-1}(a_c/a_s), & a_s < 0 \end{cases} \quad (5)$$

The cap positions, (α_1, α_2) , through which the axis passes are assumed to be at the maxima of equation 1. These are obtained from the relations:

$$\alpha_1 = k(\pi/2 - \phi) + 1 \quad (6a)$$

$$\alpha_2 = k(5\pi/2 - \phi) + 1 \quad (6b)$$

where k is $85/4\pi$, and values less than 1.0 obtained from the above relations must be added to 85.

The modulation of the sine wave, β , is computed from the ratio of the sine wave amplitude and the mean

error score. This measure, also, serves as an index of bipolarity:

$$\beta = A/M \quad (7)$$

Confidence limits may be placed on each parameter estimate by using the standard errors (SE) derived by large sample methods⁸:

$$SE(A) = (MSE/42.5)^{1/2} \quad (8)$$

$$SE(\alpha_1) = SE(\alpha_2) = kSE(A)/A \quad (9)$$

$$SE(\beta) = 0.707 SE(A)(\beta^2 + 2)^{1/2}/M \quad (10)$$

where MSE is $\sum_{i=1}^{85} (f[i] - e_i)^2/82$, and k is defined as in equation 6.

Thus, the 95% confidence interval for the axis would be $\alpha_1 \pm 1.96 SE(\alpha_1)$.

Note that the formula for the confidence interval for M has been excluded. Conventionally, a total error score (TE) is computed by subtracting 2 from each cap score, and summing over these 85 error scores. Several studies have shown that while the distribution of this total error is skewed, its square root approximates normality.^{7,9} If necessary, the square root of the total error can be computed from the mean error by the following formula:

$$TE^{1/2} = [85(M - 2)]^{1/2} \quad (11)$$

Results

Figures 1 a-d illustrate the FM 100-Hue error distributions of four color defective observers, respectively, a protan, a deutan, a tritan† and an achromat. The solid lines plot the cap scores using Kinnear's technique. The dotted circles represent reference marks signifying cap scores of 2, 5, 10, and 15, moving from the innermost to the outermost circle. The dashed line represents the fit of equation 1. Because the errors in 1d were large, the data were scaled down by a factor of three. For this case, the dotted reference circles represent cap scores of 2, 11, 26, and 41. The arrow in each plot indicates cap position 1.

The parameters estimated from each fit and the 95% confidence limits are listed in Table 1. The mean error is large for each individual. The first three observers display large amplitudes for the sinusoidal component as well. For the achromat, however, the sine wave amplitude is less than half as large. Notably, the 95% confidence interval includes zero. Another comparison can

† Dr. K. E. Higgins kindly provided the tritan data from his study of a family among whom this defect is congenital.¹⁰

Table 1. Summary of sine wave analyses of error distributions from Figure 1

Observer	Diagnosis	Mean error	Amplitude ± 1.96 SE*	Axis (cap no.) ± 1.96 SE*	Modulation ± 1.96 SE*
a	Protan	4.6	2.6 ± 0.66	(21.1, 63.6) ± 1.7	0.57 ± 0.16
b	Deutan	4.0	2.3 ± 0.71	(16.3, 58.2) ± 2.1	0.56 ± 0.19
c	Tritan	5.2	3.5 ± 0.86	(5.1, 48.8) ± 1.6	0.67 ± 0.18
d	Achromat	13.7	1.1 ± 2.02	(8.9, 51.4) ± 12.4	0.08 ± 0.15

* These confidence limits have not been corrected for autocorrelation. See text for details.

be made by examining the modulation of the sine wave. These values are listed in column 6 of Table 1. Whereas the modulation for each of the first three observers is about 0.6, indicating strong bipolarity, the modulation for the fourth observer does not differ significantly from zero.

The cap positions at which the peaks of the fitted sine waves occur are listed in Table 1, column 5. These cap positions are in accord with the center cap positions listed by others for these defects.¹ In the case of the achromat, the position of an axis has been computed, but its large confidence interval and the lack of significance of the amplitude preclude its utility.

The procedure of determining whether the confidence interval of the amplitude includes zero is useful for evaluating the statistical significance of A for an individual. However, significant autocorrelations in the residuals (differences between observed cap scores and cap scores predicted by equation 1) would indicate that the standard errors are underestimated. In fact, the autocorrelation in residuals between adjacent caps for the four observers of Figure 1 had an average of 0.48. Autocorrelation in residuals of caps displaced by more than one position, however, were not significant. Adjustment for the significant autocorrelation of adjacent residuals can be accomplished by multiplying the standard errors of each estimated parameter by the factor $(1 + 2r)^{1/2}$, where r is the estimated autocorrelation between adjacent residuals.¹¹ If the value of 0.5 is considered representative, then the confidence intervals in Table 1 should each be increased by a factor of about 1.4.

To compare axes of two individuals, form the statistic:

$$z = \frac{\alpha(1) - \alpha(2)}{(\text{SE}(\alpha(1))^2 + \text{SE}(\alpha(2))^2)^{1/2}} \quad (12)$$

where the indices refer to the axes of patients 1 and 2, respectively, from either equations 6a or 6b. This statistic can be treated as having the standard normal distribution. For the individuals of Figures 1a and 1b, z is 2.42 yielding $P < 0.02$ for a two-tailed test. In this and all further calculations, the standard errors were corrected by the factor described above.

It is important to distinguish between what is measured by the mean error and what is measured by the amplitude. The mean error is a measure of the severity of a defect. The amplitude, on the other hand, reflects bipolarity and indicates the tendency for errors to cluster on opposite sides of the error diagram. When the mean error is high, significant bipolarity usually is a sign of a dysfunction that affects a specific color mechanism in the visual system. When the mean error is low, however, a significant clustering of the error scores can still occur.

Consider the following simulation. A simple transposition of two adjacent caps in two different positions around the error diagram produces a total error score of 8 ($M = 2.09$). Let the position of one of the transpositions be held constant while the other is varied systematically. In Figure 2, the amplitude of the best fitting sine wave is plotted as a function of the position of the variable transposition. The amplitude oscillates with a frequency of two cycles. The degree of bipolarity is not significant (dashed line) when the transpositions are in adjacent quadrants. Significance is attained (solid line) when the transpositions fall in the same or op-

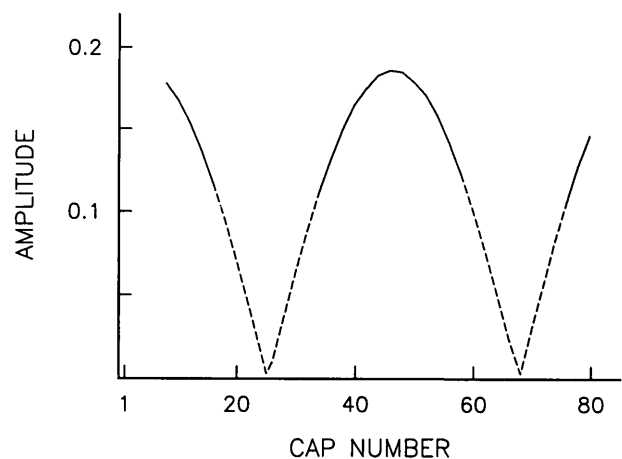


Fig. 2. Amplitude for the sine wave fits to error distributions with only two transpositions. One transposition was fixed at caps 5 and 6. The other was systematically moved about the error diagram. Dashed lines indicate amplitudes whose 95% confidence intervals include zero.

posite quadrants. At the maximum, however, the fitted sine wave has amplitude and modulation of 0.19 and 0.09, respectively.

Discussion

A method for quantifying the degree of bipolarity and the axis of FM 100-Hue error distributions has been presented. This method is similar to Kitahara's³ in that it is assumed that the amplitude and phase of a sinusoidal component of the error distribution supply this information. The method of Kitahara, however, uses an elaborate transformation of the error distribution that requires roughly 15 times as many computations as that required here. Furthermore, the normalization of the transformed error distribution that he employs is such that some individuals with only a few random errors produce patterns indistinguishable from those of individuals with severe defects.^{4,12} With the present method, low mean errors tend to generate low amplitude and low modulation sine waves. As shown in Figure 2, this does not preclude error distributions with low error rates from being significantly polar. It may ultimately be necessary to base judgments of the significance of the amplitude of the sinusoidal component on normative data as is currently done with the total error score.¹³

The current method seems to be sensitive to the presence of an axis for unipolar as well as bipolar error distributions. This is suggested by the results plotted in Figure 2 in which a statistically significant axis was detected when the transpositions fell in the same quadrant as well as in opposite quadrants.

Smith et al⁴ have proposed a different method to evaluate the axis of the error distribution. They compute the difference between red-green and blue-yellow partial error scores. This method is quick, but cannot distinguish axes differences that usually distinguish protan from deutan distributions as the present method does. This method, also, has the advantage of removing increases in the error score that are common to both red-green and blue-yellow axes. In the method presented in this paper, M and A are orthogonal parameters in equation 1. Since the minimum cap score is 2.0, the maximum values of A and β are dependent on M , however. This is the feature that insures that small mean errors yield low amplitude sine waves. We have found that the distribution across individuals of both the modulation and the square root of the amplitude are more nearly normal (unpublished observations). Note, however, from equations 8 and 9 that the variability for the amplitude and the axis does not depend on the mean error.

The method proposed here is easily programmed on the computer, and could readily be added to current

programs available for scoring of the FM 100-Hue test.^{12,14-16} The listing of a BASIC subroutine that performs the computations described above is available from the author upon request.

Note added in proof: During the period that this paper was under review, a similar approach was published in Kitahara K: A new analysis of the Farnsworth-Munsell 100-Hue test. *Nippon Ganka Gakkai Zasshi* 89:32, 1985.

Key words: color discrimination, color vision testing, Farnsworth-Munsell 100-Hue test, color deficiency

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